Bulletin of Education and Research
March 2018, Vol. 40, No. 1 pp. 231-244

Effectiveness of Problem Solving Method in Teaching Mathematics at Elementary Level

Malik Ghulam Behlol*, Rafaqat Ali Akbar** and Hifsa Sehrish***

Abstract

The study investigates effectiveness of Problem Solving Approach (PSA) in teaching mathematics to students studying at grade 8 in public schools. Pretest-posttest equivalent group design was used to conduct this study. The researcher applied matching technique to place the students in experimental and control groups (CG).Achievement test was used as a pre-test and post-test in this study. The test was validated by the experts, and table of specification was formulated to check content validity.The study revealed that the achievement level of students taught through PSA was significantly different as compared to the performance of the students taught through traditional methods of teaching on posttest. The same was the case about the performance of high and low achievers taught through PSA. Better performance of the experimental group (EG) was inferred due to active participation, self-directed learning, and higher confidence of the students in the learning process. It is also reflected that support, facilitation and guidance on the part of teacher has also contributed to the performance of the students in the EG.

Keywords: Problem solving approach, Effectiveness, Grade

* Assistant Professor, Fatima Jinnah Women, University Rawalpindi. Email: ghulam_behlol@yahoo.com
** Professor, IER, University of the Punjab, Lahore
*** Research Scholar, FJWU

Introduction

Learning is an activity or a process that engages the learner to discover, invent, self-actualize, think and apply innovative ways to solve a problem. It builds up character and conduct of students to lead a successful life. Quality of teacher education is one of important factor that contribute to the quality of education in classroom. It is for the teachers to modify and adjust their pedagogical skills to accelerate learning process in the classroom. All good teaching is characterized by proper teaching methods, and priority may be given to improve the capacities and professional competencies of the teachers to apply appropriate teaching methods that may enhance learning of students. Majority of the mathematics teachers, according to Nafees (2011) and Sherrazi (2000) follow traditional methods to teach mathematics in public schools at elementary and secondary levels in Pakistan. Consequently students by and large are not able to develop mathematical ability and interest. PSA develops and promotes mathematical ability by enhancing logical, analytical thinking skills in students to solve different problems not only in mathematics but also in daily matters of life. Every society expects from its education system to educate and train students in a way that may solve the real life problems of tomorrow (Walker & Lofton, 2003; Chin & Chia, 2004). PSA in teaching of mathematics promotes creative, analytical and interpretive skills in students to solve complicated challenges and problems (Lester & Kehle, 2003). National Council of Teachers of Mathematics (NCTM) has emphasized that PSA needs to give primary importance to the teachers who are teaching mathematics. In Pakistan, teachers working in public schools continue to use Traditional Methods of Teaching (TMT) in teaching of mathematics at grade 8. As a result of this situation, the researcher planned and designed a study to check the significance of PSA in teaching of mathematics to students studying at grade 8. It is also aimed at investigating about the effectiveness of PSA on the achievement of high and low achievers (HLAs) studying at 8^{th} grades. Results of the study may be significant for the teachers teaching mathematics to grade 8 and also for the students studying at this grade to apply PSA in the study of mathematics. It may develop positive attitude in learning of mathematics, promote conceptual understanding, enhance mathematical thinking and interests in the study of mathematics. The study was delimited to male students of grade 8 studying in Municipal Corporation Schools focusing three chapters i.e. Square Root, Percentage and Algebra of grade 8 textbook.

Objectives of the study

1. To check the effectiveness of ‘problem solving approach’ in teaching of mathematics on the achievements of students studying at grade 8.
2. To check the effectiveness of ‘problem solving approach’ in teaching of mathematics on the achievements of high achievers (HAs) studying at grade 8.
3. To check the effectiveness of ‘problem solving approach’ in teaching of mathematics on the achievements of low achievers (LAs) studying at grade 8.

Null hypotheses of the study are:

1. There is no significant difference between the achievements of the students studying mathematics at grade 8 taught by the PSA (Problem Solving Approach) and TMT (Traditional Method of Teaching) on pretest.
2. There is no significant difference between the achievements of the students studying mathematics at grade 8 taught by the PSA and TMT on posttest.
3. There is no significant difference between the achievements of HAs studying mathematics at grade 8 taught by the PSA and TMT on posttest.
4. There is no significant difference between the achievements of LAs studying mathematics at grade 8 taught by the PSA and TMT on posttest.

Literature Review

Mathematics is a discipline that promotes logical thinking and provide us tools to describe abstract ideas in quantitative terms and intelligent fashion. It contributes in the performance of daily life activities of every individual, and provides basis for the development of different subjects in natural and social sciences (Iqbal, 2004). Revolutionary development in different fields of life is direct or indirect result of mathematics. It develop accuracy, concentration, reasoning, analytical thinking, creative thinking and intellectual independence. According to a Physicist, Feynan (2002), nature can be communicated by using symbols of mathematics that helps to understand and explain the things in the universe. If a country wants to produce men and women who can create knowledge for the development and progress, then it must make sure that the proper basis may be provided at elementary and secondary schools through the study of mathematics. In this respect, mathematics teacher plays very important role, and facilitate students to think, reflect and think about thinking (Wakefield, 2001). The features for effective teaching of mathematics recognized by the Education Alliance (2006)are: 1) use experience and previous knowledge as a foundation for constructing new knowledge, 2) use cooperative learning approaches and make real-life connections, 3) use support to make connections with concepts, procedures, and understanding, 4) ask analytical questions which require students to justify their replies, 5) certify that instructional activities are learner-centered and give emphasis to inquiry/problem-solving, 6) give emphasis to the development of basic computational skills, 7) emphasize lessons on particular concept/skills that are standard-based, 8) differentiate instruction through flexible grouping, 9) modifying lessons and using tiered assignments and varying question levels.

According to Nafees (2011), problem solving is a process to solve problems through higher order cognitive operations of visualizing, associating, abstracting, comprehending, manipulating, reasoning and analyzing. PSA encourages students to promote and construct methods through practice, and reflect to solve problems (Weber, 2008). It increases self-confidence in students to think mathematically for constructing, assessing and improving their own theoretical formulas and techniques to solve problems. Teachers must be clear about what they want in their students to achieve as they structure circumstances that are both challenging and achievable for a wide range of students. Teachers need to modify the balance of control in the classroom for practicing PSA (Flowers, 1992). Teachers are required to be able to adopt instructional approaches and activities to encourage students' development of basic abilities, rational skills, and personal qualities (Crunkilton 1992; Flowers, 1992).As Weber (2008) declares that the teacher must have a solid understanding how to develop ability of arguments in students to solve a problem.

Problem based learning needs student-centered learning environment in which a student is the central figure of the learning process. The individualized, self-directed learning provides independence to the learner to decide about learning themselves under the guidance of teacher. The learning objective is not to receive the learning content without any active participation and reproducing it with memorization. It is dynamic and innovative engagement of students in group work and in individual study activities (Tick, 2007). Stepien and Gallagher (1993) have given four critical structures of problem-based learning:

1. *Engagement.* The problem addresses real matters that attribute to the larger social back ground of the students' personal world and increases values and ideas relevant to the content area.
2. *Inquiry.* It is in need of investigation to describe and improve the questions and ideas related to the problem.
3. *Solution building*. In problem-based learning, teachers are the facilitators and solutions are worked out by the students themselves. Students take part in inquiry, observation and investigation of hypotheses. They generate conclusions that are reliable and take ownership of their solutions. Teachers promote learning by acting as models/ representative behaviors they want their students to adopt.
4. *Reflection.* Assessment offer a structure of reflection as a reliable remedy to the problem, the emphasis on the difficulty of both the subject-matter concepts within the problem and cognitive process, given to perform as standards for thinking.

Van et al. (1994) has identified following characteristics of PSA:

1. Problem solving is interaction between teacher/students and vice versa.
2. The teacher helps pupils to understand and define problem clearly and he/she also endeavors to highlight importance of the problem at hand.
3. Teachers provide appropriate amount of knowledge to establish problem, and students understand, clarify, and make an attempt to formulate one or more solution procedures.
4. In a non-evaluative way teachers accept wrong/right answers.
5. Teachers need to be trained to ask perceptive questions, and play supervisory and as well as sharing role in the procedure of solving problems.
6. Teachers know when and where to step back or forward and how to let the pupils make their own way.
7. The PSA may improve problem solving skills of the students. If the students are provided opportunities to experience variety of problems besides choosing and implementing solutions, their abilities will definitely improve and they will be more likely to benefit from their problem solving ability in new situations.
8. Students may be taught to understand that there is not necessarily just a single answer to a particular question.
9. Children are often shy of speaking out and volunteering their own ideas. It is required by a teacher to facilitate and encourage students and also ask thought-provoking questions.
10. Teacher may show students how to approach a problem, formulate it and devise a strategy for its solution in addition to evaluating the problem and selecting the elements including a verbal analysis of the problem's parts which may lead them to solution.
11. In using the problem solving method, the subject matter must be organized on a basis of problem. The teacher must always be conscious of the practical value of this procedure. The material, such as references necessary for the solution of the problem, must be placed at the disposal of the pupils.
12. The teacher must bear in mind that only problems which stimulate thinking and reasoning are educative.
13. The problems should not be too broad in their scope. Many such problems make the pupils lose their interest long before a solution is reached at. In such a situation, the big problem should be divided into smaller and inter-related problems, and each small problem should be solved independently.
14. The principle of cause and effect should be emphasized while using this procedure. The development of reflective thinking is the fundamental aim of this method. The problem should involve both thinking and reasoning. Facts should be learned as part of the situation demanding reasoning and not for mere memorization purpose.

Ahmed, (2011); Albano (1996), Yusof and Tall (1995) concluded that the students taught through PSA performed better than the students taught through TMT on *achievement and problem solving ability* tests. Performance of the high, average and low achievers of EGs were better than those of the CGs. Comparison with in the EG reflects that high and average achievers of the EG performed better than the LAs on achievement and problem solving ability tests, whereas the performance of high and average achievers was equal. Reasoning ability of problem solving group was also found better than the students taught through TMT. According to Wheatley (1992), problem-centered learning encourages reflective thinking in students.

Methodology of the Study

Methodology of the study is explained under the following headings:

Design of the study

The study examines significance of PSA in teaching of mathematics to students studying at grade 8 in the Municipal Corporation School located in Rawalpindi city. Equivalent pre-test post-test group design was applied to check the effectiveness of PSA in teaching of mathematics at grade 8. This design was considered useful because it may control the internal and external validity threats to the experiment such as testing, history, maturation, instrumentation, statistical regression and experimental effects.

Table 1
Summary Sheet of the Design of the Study

Achievement Level	Experimental Group	Control Group
HAS	Block 1	Block 2
LAS	Block 3	Block 4

Population and sample of the study

The effectiveness of PSA was checked by conducting lessons and measuring performance of the students on pretest and posttest. Boys studying at grade 8 in Municipal Corporation Schools located in Rawalpindi city was the population of the study. Economically rich families send their children to private schools whereas the low economic status families send their children to public schools. The researcher applied purposive sampling technique to select the school and subjects of the study. This technique was considered useful because the researcher needed the cooperation of head of institutions to adjust the students in the control and experimental groups of grade 8, and also find out the mathematics teachers of equal experience and qualification. The researcher delivered the test to the two sections of grade 8 for the formulation of control

and experimental groups. On the basis of pretest scores, the subjects were allocated to the experimental and control groups. Matching technique was used for dividing students in experimental and control groups. The heterogeneous scores (extremely higher or low performance) that disturb the average mean of both of the groups was not included in the sample. 30 students were placed in the EG and 30 others were placed in the CG. Each group was divided into two sub-groups i.e. HAs (above the mean score) and LAs (below the mean score). Distribution of students was done on the basis of pretest scores.

Construction and validation of instrument

The researcher constructed achievement test that was used as a pretest and posttest of the study. The test consisted on objective and subjective type items. It was composed of 20 multiple-choice test items, 10 matching items, 10 completion items and 10 long questions, pertaining to a combination of knowledge, comprehension, and application level of learning outcomes. These test items were based on the following selected units of 8^{th} class mathematics: Square roots, Algebra and Percentage. The total marks of the test were 50. The content validity of the test was ensured by constructing the table of specification. The researcher also constructed a test rubric to measure the performance of the students. There was one mark for each objective type item and two and 1.5 marks for subjective type item. Construct validity of the test was maintained by obtaining the views of experts. They suggested changes, revision and deletion of the test items. Respecting their opinions, four items were deleted and six items were revised. The researcher also conducted pilot test on 20 students and discussed the ambiguities in the test items. Grammatical mistakes and conceptual ambiguities were discussed and removed in consultation with the group. In this way, the test was finalized for administration.

Development of model lessons and training of teacher

The teachers having similar qualification, training and teaching experience were not available in school. Therefore, the researcher decided to teach both of the groups himself. After studying extensive literature on PSA and getting one week training from problem solving experts, the researcher felt competent enough to conduct the classes. The steps of training according to Polya includes: *First Step: C*omprehend the problem, *Second Step:* Design a plan, *Third Step:* Execute the plan, *Fourth Step:* Looking back. For the EG, 15 lesson plans were developed on the basis of four-stage heuristic process given by Polya. Each lesson plan was divided into 6 parts. Part 1 consisted of introductory information, duration of period, name of school, topic to be taught. Part 2 was consisted of the objective and specific objectives. Part 3 consisted of establishing relationship of previous knowledge with introduction and statement of the aim. Part 4 was based on Polya's heuristic steps of problem solving (understand the problem, devise a plan, carry out the

plan and evaluate the results). Part 5 was devoted to class work and part 6 to homework. The same topics were taught to the CG by the researcher following TMT (lecture method). To control the carry over effects, the researcher taught the lessons first to the CG, and the very next day to the EG.

Duration of Experiment and Data Collection

The experiment continued for 4 weeks. Researcher taught 5 days in a week from Monday to Friday. In this way, time duration for the experimental and control groups was 20 hours each. After the completion of experiment, the achievement test that was used as pre-test was also applied as a post-test to collect data about the overall achievements of the students of EG and CG. Pretest was served to obtain base line data whereas the posttest served to measure the achievement of students as a result of teaching with the PSA. Students were consulted and taken into the confidence about the objectives of the experiment. They were willing to take part in the study. Consent of the concerning officials of the education department was obtained to conduct the experiment. The pretest, posttest scores of the experimental and control groups was the data of the study. Mean, standard deviation, standard error mean and differences of mean was computed for each group.

Data analysis

Data was analyzed under the following tables:

Table 2

Significant difference between experimental and control groups on pretest

Group	N	M	SD	SEM	t	DF	p.value	Mean difference
EG	30	18.23	3.37	.615	.420	58	.676	.367
CG	30	17.87	3.39	.619				

Table 2 shows the difference between the performance of the students placed in the EG and CG on pre-test. The results establish that there was no significant difference between the achievement levels of the two groups on pre-test. The t value was 0.420 and the p value was 0.676 that revealed that as far as achievement scores of the two groups were concerned, they had no statistical difference.

Table 3

Significance of difference between the mean scores of HAs of experimental and control groups on pretest

Group	N	M	SD	SEM	t	df	p.value	Mean difference
EG	16	20.88	2.09	.523	.591	30	.559	.438
CG	16	20.44	2.09	.524				

Table 3 reveals the difference between the performance of the HAs placed in the EG and CG on pre-test. The results establish that there was no significant difference between the achievement levels of the HAs of the two groups on pre-test. The t value was .591 and the p value was .559. It revealed that as far as achievement scores of the two groups were concerned, they had no significant difference.

Table 4

Significance of difference between mean scores of LAs of experimental and control groups on pretest

Group	N	M	SD	SEM	T	df	p.value	Mean Difference
Experimental Group	14	15.21	1.36	.366	.681	26	.502	.357
Control Group	14	14.86	1.40	.376				

Table 4 reveals the difference between the performance of the LAs placed in the EG and CG on pre-test. The results establish that there was no significant difference between the achievement levels of the LAs of the two groups on pre-test. The t value was 0.681 and the p value was 0.502 that revealed that as far as achievement scores of the two groups were concerned, they had no significant difference.

Table 5

Significance of difference between mean scores of experimental and control groups on posttest

Group	N	M	SD	SEM	t	df	p.value	Mean difference
EG	30	34.90	5.03	.919	12.69	58	.000	14.30
CG	30	20.60	3.56	.651				

Table 5 reveals the difference between the performance of the students placed in the EG and CG on post-test. The results establish that there was significant difference between the achievement levels of the two groups on post-test. The t value was 12.69 and the p value was 0.000 that revealed that as far as achievement scores of the two groups were concerned; they had significant difference. It was inferred that the students who were in the EG performed better due to active, logical and student centered involvement in the learning process originated by the PSA.

Table 6

Significance of difference between the mean scores of the HAs of experimental and control groups on posttest

Group	N	M	SD	SEM	T	df	p.value	Mean Difference
Experimental Group	16	33.94	5.19	1.29	7.84	30	0.000	11.18
Control Group	16	22.75	2.35	.588				

Table 6 reveals the difference between the performance of the HAs placed in the EG and CG on post-test. The results establish that there was significant difference between the achievement levels of the HAs of the two groups on post-test. The *t* value was 7.84 and the p value was .000. It revealed that the achievement scores of the HAs of the two groups were significant difference. It was inferred that the HAs who were in the EG performed better due to the greater frequency of interaction between peer and teacher-student, relevance to real life situation, individualized engagement in the learning process originated by the PSA.

Table 7

Significance of difference between the mean score of the LAs of experimental and control groups on posttest

Group	N	M	SD	SEM	t	Df	p.value	Mean Difference
EG	14	36.00	4.78	1.28	11.67	26	.000	17.85
CG	14	18.40	3.13	.838				

Table 7 reveals the difference between the performance of the LAs placed in the EG and CG on post-test. The results establish that there was significant difference between the achievement levels of the LAs of the two groups on post-test. The t value was 11.67 and the p value was .000. It revealed that as far as achievement scores of the LAs of the two groups were concerned, they had significant difference. It was inferred that the LAs who were in the EG performed better due to the confidence and interaction of high and low achievers in the learning process originated by the PSA.

Discussion and Conclusions

Different factors influence students' achievement scores on academic test. The current study investigated two methods of teaching: PSA and TMT. The major aspects of the study was to check the effectiveness of problem solving approach in teaching of mathematics on students' academic performance enrolled in Municipal Corporation secondary schools of Rawalpindi. Students who were taught through PSA performed better as compared to those students who were taught through TMT. This significant

difference was inferred due to the involvement and interest in the learning process generated with the support of PSA. It may provide hands-on experiences to students in learning mathematical concepts that ensured the active involvement of the students in learning process. Hence, it was established that achievement score of EG was significantly different than the achievement score of the CG on posttest. The result of the study was confirmed by the findings of the study conducted by Khan, Akhter & Hukamdad (2010).

HAs and LAs also excelled on the post-test as compared to the students of the CG taught by the TMT. These results support the findings of the study conducted by Kousar (2009) on the effectiveness of problem solving method. Better performance of the HAs was due to student centered learning approach that has provided opportunities to students to think, reflect and apply the solution of the problem in learning tasks. It may help them think about the solutions of the problems they may face in real life also. It is inferred that the better performance was due to reorganization of information and reconstruction of arguments by the students themselves. The results were also supported by studies conducted by Hsiao & Chang, (2003); Tang &Huang (2006). They found significant difference may be deduced to the active participation, raising level of confidence in the students of EG. The difference in performance may occur due to reflective thinking approach.

It was inferred from data analysis that students taught by PSA retained the subject matter more due to greater understanding than the students taught by traditional method. On the whole, it was concluded that the PSA was found to be cognitively effective in achieving learning outcomes. It is more helpful for high and low achievers of mathematics students. It allows students to work according to their capabilities and make decisions to explore solutions to the problem. Problem solving approach sets its focus on the student making sense of mathematical ideas.

Recommendations

Following were the recommendations of the study:

Mathematics teachers may be trained to apply PSA in teaching mathematics at elementary level. It is also recommended to develop lesson plans according to the basic principles of problem solving method.

Textbook writers may incorporate problem solving activities in textbooks to facilitate the teaching learning process on the basis of the problem solving teaching strategy.

It is also recommended that the heads of the institutions may facilitate the teachers to adjust the classroom environment to apply problem solving method.

Findings and conclusions of the present study suggested that problem solving method is effective in teaching of mathematics at elementary level. However, there is a need of some more studies to explore the effectiveness of PSA at elementary level on different population and subjects.

References

Ahmad, M. (2011). *Effect of problem solving teaching strategy on the performance of 8th grade students.* PhD unpublished thesis, University of the Punjab, Lahore.

Albano, M. G., Cavallo, F., & Hoogenboom, R., (1996). An international comparison of knowledge levels of medical students: the Maastricht Progress Test, *Journal of Medical Education*, 239-245.

Alliance, E. (2006). Closing the achievement gap: *Best practices in teaching.* Mathematics Charleston, WV: The Education Alliance.

Chin, C. & Chia, L. G. (2004). Problem-based learning: Using students' questions to drive knowledge construction, *Journal of Science Education,88(5).*

Crunkilton, J. R. (1992). SCANS report and problem solving: A natural alliance. *Agricultural Education Magazine 65, no. 5*.

Dyer, J. E., & Osborn, E. W. (1996). Effects of teaching approach on problem solving ability of agricultural education students with varying learning styles. *Journal of Agricultural Education, 37,* No. 4, 1996. Retrieved from pubs.aged.tamu.edu/ jae/pdf/vol37/37-04-36.pdf.

Feynan, R. (2003). *The new book of popular science.* Danbury, Conn: Grolier.p.328.

Flowers, J. (1992). Problem solving instruction: Making students gladiators instead of spectators. *Agricultural Education Magazine 65*(5).

Hoffer, T. B., & Gamoran, A. (1993). *Effects of instructional differences among ability groups on student achievment in middle-school science and mathematics.* Report Center on Organization and Restructuring of Schools.University of Wisconsin, Madison, WI.P.51.

Hsiao, H. C., & Chang, J. C. (2003). A quasi-experimental study researching how a problem solving teaching strategy impacts on learning outcomes for engineering students. *World Transactions on Engineering & Technology Education. UICEE 2*(3). Retrieved from http://www.eng.monash.edu.au.

Iqbal, M. (2004). *Effect of cooperative learning in academic achievement of secondary school students in mathematics* (Unpublished P.hD. thesis) University of Arid Agriculture Rawalpindi (Pakistan) PP. 2-3.

Khan, A., Akhter, A, & Hukmadad. (2010). Asian school science. *Effect of using problem solving method in teaching mathematics on the achievement of mathematics Students. 6.*

Kousar, P. (2010). Effect of the problem-solving approach on academic achievement of students In mathematics at the secondary level. *Contemporary Issues in Education Research.*

Lester, F. K., & Kehle, P. E. (2003). From problem solving to modeling: The evolution of thinking about research on complex mathematical activity. *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*, 501-518.

Ministry of Education (2006). *National curriculum for mathematics grades I-XII,2006*. Government of Pakistan Ministry of Education, Islamabad.

Nafees, M. (2011). *An experimental study on the effectiveness of problem-based versus lecture-based instructional strategy on achievement, retention and problem solving capabilities in secondary school general science students.* PhD unpublished thesis, International Islamic University, Islamabad.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for School mathmatics*, Reston, Virginia, NcTM. P.77.

Serieux, J. (2000). Mathematics introduction to revised curriculum for mathematics education. Retrieved from www.eduactiongov./c/edu.com/mathematics.htmal.

Sherrazi, S. A. (2000). *Re-orientation of mathematic teaching.* Retrieved from http://prr.hec.gov.pk/Thesis/2654H.pdf on 24.12.13

Stepin, W. J., & Gallagher, S. A. (1993). Problem-based learning: As authentic as it gets. *Educational leadership*, 50(7).

Tang, j., & Huang, M. (2006). An application of problem-solving model on the auto mechanics practicum course in vocational high schools *Jouranal of Eductaion and Psycholoy*, 29*(3).* Retrived from www.fed.cuhk.edu.hk\en\jep.

Tick, A. (2007). Application of Problem-Based Learning in classrooms activities and multimedia. *5th Slovakian Hungarian Joint Symposium on Applied Machine Intelligence and Informatics.* Retrieved from http://bmf.hu/conferences/ sami2007/36_Andrea.pdf

Van, Z, L., G. Jones C. & Thoration. (1994). Beliefs about mathematics teaching hold by pre-service teachings involved in first grade mentorship program. *Mathematics Education Research Jorunal 6(1).*

Wakefield, A. P. (2001). Teaching young children to think about math. *Principal, 80*(5), 26-29.

Walker, J. T., & Lofton, S. P. (2003). Effect of a problem based learning curriculum on students' perceptions of self directed learning. *Issues in Educational Research, 13*.

Weber, K. (2008). Mathematicians validation of proofs. *Journal for research in mathematics education. 39(4),* p.432.

Wheatly, G. H. (1992). The role of reflection in mathematics. *Educational studies in mathematic*. *23(5).*

Yusof, M, Y. & Tall, D. O. (1995). Professors' *perceptions of students' mathematical thinking*: Do They Get What They Prefer or What They Expect? In L. Meira &D. Carraher, (Eds.), *Proceedings of PME* 19, II,pp, 170–177